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MATLAB-BASED ALGORITHM TO SOLVING AN OPTIMAL STABILIZATION PROBLEM FOR THE DESCRIPTOR SYSTEMS

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Abstract In the work solution of the optimal stabilization problem, when movement of the object is described by the descriptor system is considered using MATLAB package procedures. A numerical algorithm is developed for the solution of this problem. The obtained results are illustrated on the example.

Key words: Descriptor systems, output feedback, linear system, closed- loop system.

AMS Mathematics Subject Classification: 65K15.

1 Introduction

We propose a general, numerically reliable computational approach to solve the optimal stabilization problem for the descriptor systems. This study is stimulated by strong relations of these problems to some technical and practical systems, as well as mobile manipulators, electrical networks and etc. [13,14]. Considering the recent achievements of the computational techniques it becomes important development of the new methods which allow one to cover larger classes of the practical problems, raise the accuracy and reduce the computational costs [16]. On the base of Levine-Athans [1] algorithm is shown that the proposed algorithm requires solution of two generalized algebraic Lyapunov equation [2], which is solved by signium function and orthogonal projections methods. Initial value is chosen from the asymptotical stability condition of the closed-loop system [10].

Note that the similar problem for the non-descriptor systems has been considered by various authors. In [1] the solution of such problems are reduced to the solution of three non-linear equations. In [3-5] the methods of convex analysis are applied to construct the solution of the problem. In the works [6,11] the fine function method is used.

But a wide class of practical problems is described by the so called descriptor systems when the determinant of the coefficient matrix is equal to zero. The paper deals with this case.

2 Problem formulation

Let the movement of the object be described by the system of linear differential equations with constant coefficients

$$E\,\dot{x}(t) = Fx(t) + Gu(t); \quad E\,x(0) = X_0, \tag{1}$$

$$y = Cx\,(t)\,.$$

It needs to minimizes the functional

$$J = \int_0^\infty (x'Qx + u'Ru)dt \tag{2}$$

by the regulator low

$$u\,(t) = Ky\,(t) \tag{3}$$

under the condition that close-loop system (1),(3) was asymptotically stable.
Here $x \in R^n$-is a phase vector of the object coordinates, $u \in R^m$- control vector, $y \in R^z$- observable vector, $E-(n\times n)$, $F-(n\times n)$, $G-(n\times m)$, $C-(z\times n)$, $Q-(n\times n) = Q' \geq 0$, $R-(m\times m) = R' > 0$ are given constant matrices. Suppose that the matrix E is singular, i.e. $rankE = r < n$. In this case (1) is call a descriptor system.

Considering (3) in (1) we obtain the problem

$$E\,\dot{x}(t) = (F + GKC)x(t); \;\; E\,x(0) = X_0. \tag{4}$$

To find the optimal K this system must be strongly stable.
Definition. System (4) is called strongly stabilizable if

$$rank(sE - F - GKC) = n$$

for any complex s with non-negative real part [10].

First suppose that $\det(E) \neq 0$. It is known [9] that in this case the solution of the regularized problem is close enough to the solution of the considered non-regularized problem. Then solution of the system (1) may be written in the following form

$$\dot{x}(t) = \bar{F}x(t) + \bar{G}u(t)\;,\; x(0) = E^{-1}X_0, \tag{5}$$

where

$$\bar{F} = E^{-1}F\;;\;\; \bar{G} = E^{-1}G. \tag{6}$$

According to [1, 7] solution of the problem (5), (2), (3) is reduced to the solution of the equation with respect to S and U correspondingly

$$(\bar{F} + \bar{G}\bar{K}C)'S + S(\bar{F} + \bar{G}\bar{K}C) + Q + C'\bar{K}'R\bar{K}C = 0, \tag{7}$$

$$(\bar{F} + \bar{G}\bar{K}C)U + U(\bar{F} + \bar{G}\bar{K}C)' + X_0 = 0, \tag{8}$$

$$\bar{K} = -R^{-1}\bar{G}'SUC'(CUC')^{-1}. \tag{9}$$

Considering (6) after some technical transformations the system (7)-(9) may be written in the form

$$(F + GKC)'PE + E'P(F + GKC) + Q + C'K'RKC = 0, \tag{10}$$

$$(F + GKC)UE' + EU(F + GKC)' + EX_0E' = 0, \tag{11}$$

$$K = -R^{-1}G'PEUC'(CUC')^{-1}, \tag{12}$$

where

$$P = E'^{-1}\, S\, E^{-1}. \tag{13}$$

As is known there exist calculation algorithms for the solution of the problem (5),(11) that require to solve two algebraic Lyapunov equations in each step. Now we try to generalize this result for the system (10)-(12). In step 2 of algorithm 1 below we do this using the corresponding MATLAB procedures. In this case the generalized Lyapunov equation must be solved in each step. In defer from (7)-(9) in (10)-(12) we have E instead of E^{-1}. This fact may make easy the calculation procedure. Thus the following algorithm may be offered for the solution of the equations (10)-(12).

Algorithm 1

1. Choose initial approach K_i such that eigenvalues of the matrix $(E - F - GK_iC)$ were negative.
2. Solve the generalized algebraic Lyapunov equations (10)-(11).
3. Calculate

$$K_{i+1} = -R^{-1}G'P_iEU_iC'(CU_iC')^{-1};$$

4. Check up the condition $\|K_{i+1} - K_i\| \leq \varepsilon$. If it is satisfied then the process is over. Otherwise taking $K_i = K_{i+1}$ go to step 2.

Note that the convergence of this algorithms is studied in [10].

Now let's formulate discrete linear quadratic problem similarly to continuous case.

Let the object's motion be described by the stationary system of finite – difference equations

$$Ex(i+1) = \Psi x\,(i) + \Gamma u\,(i)\,, \qquad i = 1, 2, ..., Ex(0) = X_0 \tag{14}$$

$$y\,(i) = Cx\,(i)\,,$$

where $\Psi = e^{F\triangle}$, $\Gamma = (e^{F\triangle} - E)F^{-1}G$, $\triangle$ is a discretization interval.

It needs to minimized the functional

$$J = \sum_{i=0}^{\infty} \left(x'\,(i)\,Qx\,(i) + u'\,(i)\,Ru\,(i)\right), \tag{15}$$

by the regulator low

$$u(i) = Fy(i) = FCx(i), \tag{16}$$

under the condition that close-loop system (14)-(16) was asymptotically stable. E ill a conditioned matrix

It is supposed that $\det(E) \neq 0$. Then solution of the system (14) may be written in the following form

$$x(i+1) = E^{-1}\Psi x\,(i) + E^{-1}\Gamma u\,(i)\,, \qquad i = 1, 2, ..., x(0) = E^{-1}X_0. \tag{17}$$

Solution on the problem (17),(15),(16) is reduced to the solution of the equations [8,9]

$$L = \left(\bar{\Psi} + \bar{\Gamma}FC\right)'\, L\left(\bar{\Psi} + \bar{\Gamma}FC\right) + Q + C'FRFC, \tag{18}$$

$$U = \left(\bar{\Psi} + \bar{\Gamma} FC\right) U \left(\bar{\Psi} + \bar{\Gamma} FC\right)^{'} + X_0 \ , \tag{19}$$

$$F = -\left(R + \bar{\Gamma}' L \bar{\Gamma}\right)^{-1} \bar{\Gamma}' L \bar{\Psi} U C' \left(C U C'\right)^{-1} , \tag{20}$$

where

$$\bar{\Psi} = E^{-1}\Psi \ ; \bar{\Gamma} = E^{-1}\Gamma. \tag{21}$$

If consider the definition (21) then the system (18)-(20) one can write in the form

$$E'PE = (\Psi + \Gamma FC)^{'} P (\Psi + \Gamma FC) + Q + C'FRFC, \tag{22}$$

$$EUE' = (\Psi + \Gamma FC) U (\Psi + \Gamma FC)^{'} + X_0 \ , \tag{23}$$

$$F = -(R + \Gamma' P \Gamma)^{-1} \Gamma' P \Psi U C' (CUC')^{-1} , \tag{24}$$

where

$$P = E'^{-1} L E'.$$

Similarly to continuous case in defer from (17)-(19) we have E instead of E^{-1}in (22)-(24).

This fact may make easy the calculation procedure. Thus one may offer the following algorithm for the solution of the equations (22)-(24).

Algorithm 2.

1. Chose initial approach F_i such that the eigenvalues of the matrix $(E - (\Psi + \Gamma F_i C))$ was inside of the unit circle;
2. Solve the following generalized discrete algebraic Lyapynov equation

$$E' P_i E = (\Psi + \Gamma F_i C)^{'} P_i (\Psi + \Gamma F_i C) + Q + C' F_i R F_i C,$$

$$E U_i E' = (\Psi + \Gamma F_i C) U_i (\Psi + \Gamma F_i C)^{'} + E X_i E;$$

3. Calculate F_{i+1}

$$F_{i+1} = -(R + \Gamma' P_i \Gamma)^{-1} \Gamma' P_i \Psi U_i C' (C U_i C')^{-1} ;$$

4. Check up the condition $\|F_{i+1} - F_i\| \leq \varepsilon$. If it is satisfied then the process is over. Otherwise taking $F_i = F_{i+1}$ go to step 2.

For illustrating the offered algorithm, we consider the following examples from [1].

Example. [1] Values of the matrices in (1),(2), $E, F, G, C,\ Q,\ R$ are as follows

$$E = \begin{bmatrix} 1 & 0 \\ 0 & a \end{bmatrix} ; \ G = \ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \ ; C = \begin{bmatrix} 0 & 1 \end{bmatrix} ;$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \ ; R = 1 ; \ F = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} .$$

The calculations are carried out for different values of a. The solution K of the problem (7)-(9) obtained by the method given in [1] we define by K_{ath} and the corresponding coefficient of the optimal regulator by G_{ath}. The same results obtained by solving the equations (10)-(12) using the above mentioned algorithm we denote by K_d and G_d.

Table 1: The order of approximation of the gradient for the different values of a.

a	K_{ath}	K_d	G_{ath}	G_d
1	-0.8165	-0.8165	0.000543e^{-15}	0.000543e^{-15}
10^{-6}	-7.071e^{-4}	-7.07106e^{-4}	1.995e^{-7}	1.24e^{-8}
10^{-7}	-2.2361e^{-4}	-2.2361e^{-4}	2.3772e^{-5}	2.6233e^{-8}
10^{-8}	-7.07107e^{-5}	-7.07106e^{-5}	2.5408e^{-5}	4.2534e^{-7}
10^{-9}	-2.2361e^{-5}	-2.2361e^{-5}	1.6221e^{-5}	7.3852e^{-10}
10^{-10}	-7.07108e^{-6}	-7.0725e^{-6}	9.3583e^{-4}	2.1024e^{-7}

In table below the order of approximation of the gradient for the different values of a.

Comparing the corresponding values of G_{ath} and G_d for the different values of a we see that G_d is less that G_{ath} on few orders. This fact demonstrates the efficiency of the proposed algorithm.

Note. Using the recent development of the high accuracy calculation techniques (as well as Symbolic calculation methods) [12] the proposed algorithms may be modified and give high accuracy results.

3 Conclusion

The paper is devoted to the development the algorithms to the solution of the optimal stabilization problems for the descriptor systems. The results of the numerical experiments for the test examples are given.

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References

[1] Levine, W.S., Athans, M., *On the determination of the optimal constant output feedback gains for linear multivariable systems*, IEEE Trans. Autom. Control. Vol.AC-15, No.1, (1970).

[2] Boyd, S., Yang, Q., *Structured and simultaneous Lyapunov functions for system stability problems*, Int. J. Contr., Vol.49, No.6, , pp.2215-2240, (1989).

[3] Peres, P.L.D., Geromel, J.C. *An alternate numerical solution to the linear quadratic problem* IEEE Trans. Autom. Control, Vol.39, No.1, (1994).

[4] Bernussou, J., Peres, P.L.D., Geromel, J.C. *A linear programming oriented procedure for quadratic stabilization of uncertain systems*, Syst. Contr. Lett., Vol.13, pp.65-72, (1989).

[5] Geromel, J.C., Peres, P.L.D., Bernussou, J. *On a convex parameter space method for linear control design of uncertain systems*, SIAM J. Contr. Optimz., Vol.29, No.2, 381-402, (1991).

[6] Aliev, F.A., Velieva, N.I., Mardanov, M.D. *An algorithm for solving the synthesis problem for an optimal stabilization system with respect to output variable*, Engineering Simulat., Vol.13, 625-634, (1996).

[7] Larin, V.B. *Stabilization of the system by static output feedback*, Appl. Comput. Math., Vol.2, No.1, 47-51, (2003).

[8] Afanas'ev, A.P., Dzyuba, S.M., Emelyanova, I.I., Ramazanov, A.B., *Optimal control with feedback of some class of nonlinear systems via quadratic criteria*, Appl. Comput. Math., Vol.15, No.1, 78-87, (2016).

[9] Fridman, E. *A descriptor system approach to nonlinear singularly perturbed optimal control problem*, Automatica, Vol.37, No.4, 543-549, (2001).

[10] Aliev, F.A., Larin, V.B. *Stabilization problems for a system with output feedback (review).* International Applied Mechanics. Vol.47, No.3, pp.225-267.

[11] Safarova, N.A., Velieva, N.I. *Iterative algorithms to the solution of the discrete optimal regulator problem*, Bull. Math. Soc. Sci. Math. Roumanie, Vol.57, No.4, 427-436, (2014).

[12] Aliev, F.A., Velieva, N.I., Gasimov, Y.S., Safarova, N.A., Agamalieva, L.F. *High-accuracy algorithms to the solution of the optimal output feedback problem for the linear systems*, Proceedings of the Romanian Academy, Series A, Vol.13, No.3, 207–214, (2012).

[13] Masubuchi, Izumi, *Dissipativity inequalities for continuous-time descriptor systems with applications to synthesis of control gains*, Systems & Control Letters, Vol.55, No.2, 158-164, (2006).

[14] Bunse-Gerstner, Angelika, et al., *Feedback design for regularizing descriptor systems. Linear algebra and its applications*, Vol.299, No.1-3, 119-151, (1999).

[15] Gorbachenko, V.I. *Calculation Linear Algebra with Examples in MATLAB*, BHV-Petersburg, 320p. http://www.bhv.ru/books/full contents.php?id=189162),(2011)

[16] Karchevsky A.L., *Reformulation of the inverse problem statement that reduces computational costs*, Eurasian Journal of Mathematical and Computer Applications, Vol.1, No.2, 4-20, (2013).

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