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APPLICATION OF REVERSE TIME MIGRATION (RTM) PROCEDURE IN ULTRASOUND TOMOGRAPHY, NUMERICAL MODELING.

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Abstract This work is devoted to the medical ultrasound imaging problem. We present some results of numerical modeling, using Reverse Time Migration procedure. In the numerical experiment we use acoustical model that simulates cross-section of breast containing fat, glandular tissue and weak inclusions with a radius from $0.8mm$. It is impossible to determine these inclusions on the image obtained after a standard procedure RTM with a constant speed of sound. We use an approach based on a modified version of RTM to determine unknown boundary separating fat and glandular tissue and weak inclusion. The final image contains all the inhomogeneities and close inclusions are separated at that.

Key words: ultrasound medical tomography, Reverse Time Migration, numerical modeling

AMS Mathematics Subject Classification: 65N06, 65N21, 92C55

1 Introduction

We consider the medical ultrasound imaging problem for diagnostics of breast cancer. Nowadays groups of scientists in Russia (MSU, Department of Acoustics [1]), in the USA (N. Duric, C. Li, P. Littrup, S. Schmindt, etc.[2]) and in Germany (N. Ruiter, R. Dapp, M. Zapf, R. Jirik, I. Peterlik, J. Fousek, etc. [3]) are working on development of models of the ultrasonic tomographs with high resolution and informativity. It is well-known that ultrasound imaging has a great potential for the detection and diagnostics of breast cancer [4]. However, design of effective algorithms for measurements processing is still open.

In modern models of ultrasound tomographs 256 transducers (V.A. Burov, N. Duric) and more are used. Transducers are located on a circle with a radius from 10 cm. The dominant frequency of the impulse is from 1 to 8 MHz. Ultrasonic medical diagnostics requires a high resolution from 0.1 mm. As a result, it is necessary to work with millionth order grids. When we deal with such a great amount of data, we can try a popular procedure in Geophysics known as Reverse Time Migration (RTM). This procedure does not solve the inverse problem for the acoustic equation but can give an image of inhomogeneities of acoustical medium.

2 Formulation of the problem

Let 2-dimentional real space R^2 is filled with inhomogeneous acoustical medium. We assume that speed of sound is constant outside the disc Ω. Transducers are located at the circle $\Gamma = \partial\Omega$. A fixed source emits an ultrasound impulse at $t = 0$, that initiates a wave. The receivers measure the pressure from each source independently (Fig. 1).

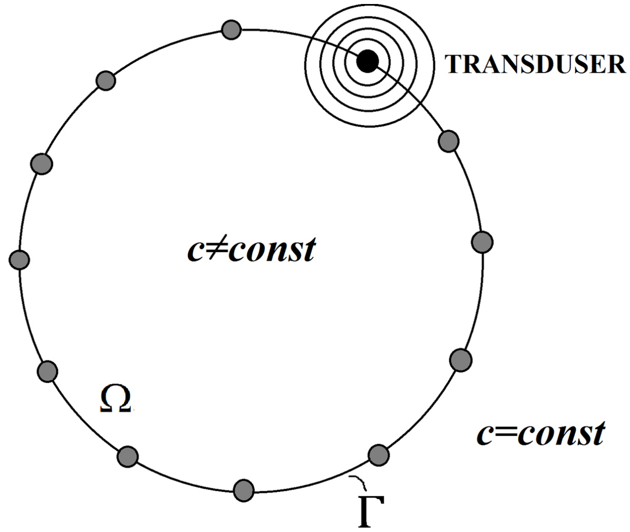


Figure 1: The scheme of the experiment

The problem is to find the image of inhomogeneities in the disc Ω from the given boundary measurements. The registration time must be big enough to order view all inhomogeneities. Let T^* is the «acoustical» radius of disc Ω, i. e. minimal time that is required to fill disc Ω with waves initiated by all boundary sources (filling time). Since the measurements registered at circle Γ must contain information about inhomogeneities in the whole disc Ω, then, as it follows from simple kinematic argument, registration time should be not less than $2T^*$. So we will assume that the inequality $T > 2T^*$ is fulfilled. Notice, that the time T^* always can be estimated by the lower bound of speed.

3 Forward problem

Consider the Cauchy problem for the wave equation in R^2 (forward problem).

$$\frac{1}{c^2} p_{tt} - \Delta p = \delta(x - x_s) r(t) \ in \ \mathrm{R}^2 \times (0, T) \tag{1}$$

$$p|_{t=0} = p_t|_{t=0} = 0 \ in \ \mathrm{R}^2 \tag{2}$$

Here $p(x, t; x_s)$ is the solution of the forward problem (1)-(2) (pressure), $\delta(x - x_s)$ is the Dirac function which models point source located at point x_s on the circle $\Gamma = \{x \in \mathrm{R}^2 |\ |x| = R\}$, $r(t)$ is the Ricker impulse (fig. 2), c is a smooth strictly positive function (speed of sound) that is equal to constant outside the disc Ω, $c(x) = const, \ x \in \mathrm{R}^2 \setminus \Omega$.

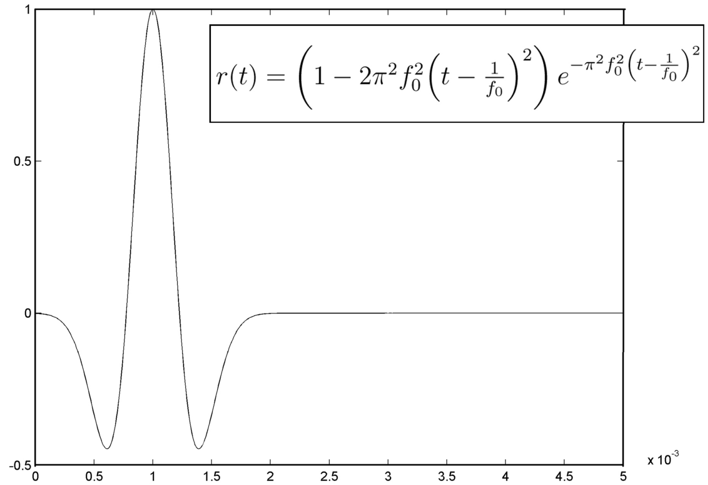


Figure 2: Ricker impulse (f_0 - dominant frequency, $r|_{t\leq 0} = 0$)

4 Imaging problem

Let us consider the inverse problem of image reconstruction of the sound speed by the boundary pressure measurements (scattered wavefield) $p_0(x,t;x_s) = p(x,t;x_s)$, $x, x_s \in \Gamma, t \in [0,T]$, $T > 2T^*, T^* = sup_{x\in\Omega}\ dist(x,\Gamma)$ for different positions of sources.

We assumed the speed of sound in the fatty and glandular tissues is known. Notice, that these speeds are known approximately from medical data. We would like to emphasize that the boundary between fat and glandular tissues is unknown as inclusions as well.

5 Algorithm of RTM

After substitution $u = p_t$, $v = \nabla p$, write down the problem (1) as equivalent first-order system

$$\begin{cases} \frac{1}{c^2}\, u_t = div\ v + \delta(x - x_s)\, r(t), \\ \qquad\qquad v_t = \nabla u. \end{cases} \tag{3}$$

One version of the well-known geophysical method of RTM, which is also called the Energy RTM, consists of the following steps [5](notice, that all migration methods need the knowledge of speed):

Step 1. First we find a pair $u^{for}(x,t;x_s),\ v^{for}(x,t;x_s)$, solving the following Cauchy problem with some known speed of sound c_0:

$$\begin{cases} \frac{1}{{c_0}^2}\, u^{for}_t = div\ v^{for} + \delta(x - x_s)\, r(t), \\ \\ v^{for}_t = \nabla u^{for}, \\ \\ u^{for}|_{t=0} = 0,\ u^{for}_t|_{t=0} = 0,\ t \in (0,T). \end{cases}$$

Step 2. We get a pair $u^{back}(x,t;x_s)$, $v^{back}(x,t;x_s)$ by time reversal continuation of the registered wavefield $u_0(x,t;x_s) = \frac{\partial p_0}{\partial t}(x,t;x_s)$ into acoustical medium with the same c_0. Mathematical problem looks as follows:

$$\begin{cases} \frac{1}{c_0{}^2}\, u_t^{back} = div\ v^{back}, \\ v_t^{back} = \nabla u^{back} + u_0\delta(|x|-R)\nu_\Gamma, \\ u^{back}|_{t=T} = 0,\ u_t^{back}|_{t=T} = 0, \end{cases} \tag{4}$$

where ν_Γ is the outward normal to the boundary Γ, $t \in (0,T)$.

Step 3: We use the Imaging condition from Energy RTM [5]:

$$I_E(x) = \sum_{x_s} \int_0^T [u^{for}u^{back} + (v^{for}, v^{back})](x,t)dt. \tag{5}$$

.

Remark. *In the standard RTM Imaging condition has no the second component in the integrand in (5).*

6 Numerical Solution of the forward problem

Numerical solution of the first order system of acoustics equations (3) is based on the explicit conditionally convergent finite-difference scheme with shifted grids (u is calculated in integer nodes in space and in half-integer nodes in time, v is calculated in integer nodes in time and in half-integer nodes in space x and integer nodes in space y) [6]. Finite-difference scheme is written as follows:

$$\begin{gathered} \left(\frac{1}{c^2}\right)^{i,j} \frac{u_{i,j}^{k+1/2} - u_{i,j}^{k-1/2}}{\Delta t} = (D_x v_1)_{i,j}^k + (D_y v_2)_{i,j}^k + r_k \delta_{i_s j_s}, \\ \frac{(v_1)_{i+1/2,j}^{k+1} - (v_1)_{i+1/2,j}^k}{\Delta t} = (D_x u)_{i,j}^{k+1/2}, \\ \frac{(v_2)_{i,j+1/2}^{k+1} - (v_2)_{i,j+1/2}^k}{\Delta t} = (D_y u)_{i,j}^{k+1/2}, \end{gathered} \tag{6}$$

where

$$\delta_{i_s j_s} = \begin{cases} 1, i = i_s, j = j_s \\ 0, otherwise \end{cases}.$$

To reduce the numerical dispersion the spatial derivatives are approximated with 12^{th} order:

$$(D_x f)_{i,j} = \frac{1}{\Delta h} \sum_{k=1}^{6} a_k (f_{i+(k-1/2),j} - f_{i-(k-1/2),j}) + O(\Delta h^{12})$$
$$(D_y f)_{i,j} = \frac{1}{\Delta h} \sum_{k=1}^{6} a_k (f_{i,j+(k-1/2)} - f_{i,j-(k-1/2)}) + O(\Delta h^{12})$$

Steps in space and time are selected according to the Courant condition:

$$\Delta t < \frac{\Delta h}{k\ c_{max}\sqrt{2}}$$

where the coefficient k depends on the order of approximation (for 12^{th} order it is about 1.34, [7]).

Since the function c is a constant outside of disc Ω, in order to limit the computational domain it is naturally to use a known technique of perfectly matched layers (PML). The computational domain was bordered by an absorption layer with specially chosen parameters in order to attenuate waves reflected from the outer boundary [8].

7 Numerical experiments

Numerical experiments were fulfilled with the following parameters (the experimental scheme is shown in Fig 3.):

Computational domain is a square $0.32 \times 0.32\ m$;
Transducers are located on the circle of radius $r = 0.15m$;
The number of nodes is 10 240 000;
The number of transducers is 256;
Step in space is 0.0001 m;
Width of PML layer is 0.01 m;
Step in time is 0.00001 ms;
Dominant frequency of Ricker's s impulse is 1 MHz.

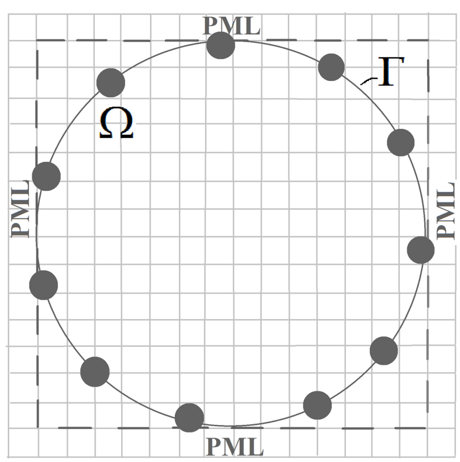


Figure 3: The scheme of the experiment

We used the acoustical model close to actual medical problem (Fig. 4) [9]. The model has a 8 inclusions with a radius from 0.8 mm to 3 mm. The values of the speed of sound is close to the real speed in the tissues: $c_0 = 1.515\ m/ms$ background speed of sound (glandular tissue), the speed of inclusion from 1.417 m/ms (fat) to 1.559 m/ms (tumors). The boundary between fat and glandular tissues is denoted by «G».

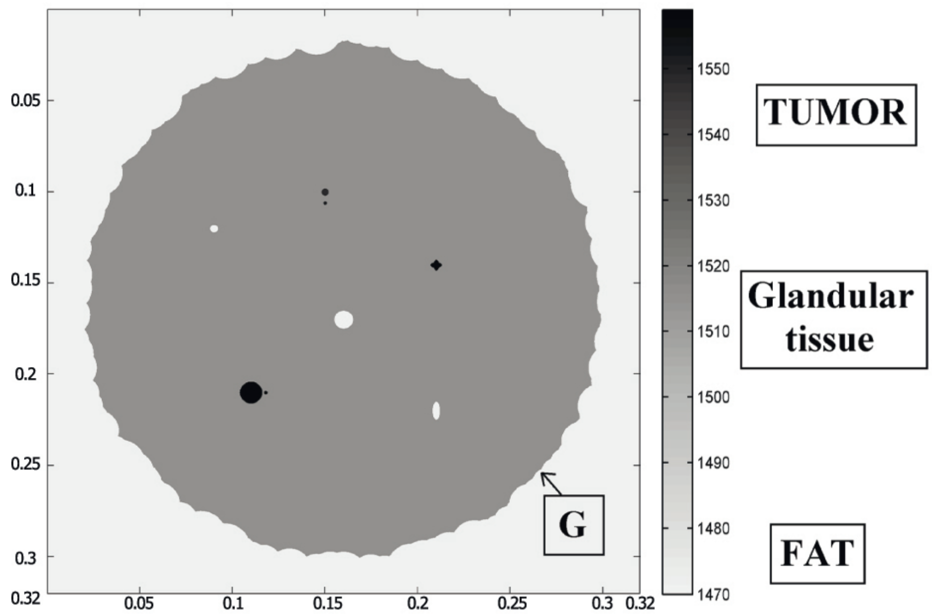
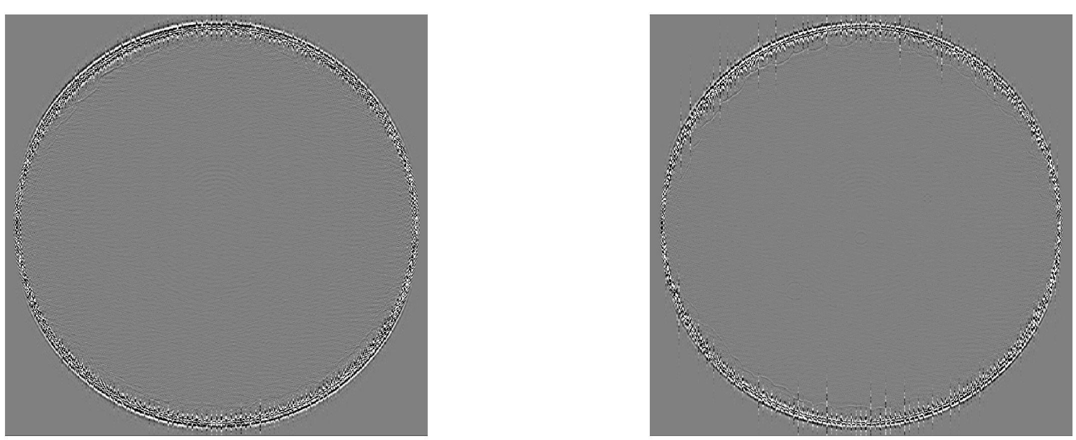


Figure 4: The model of the speed

Using the finite-difference method (6) the wave field u_0 for this model (Fig. 4) was calculated. Then the standard version of RTM and Energy RTM were applied ($c_0 = 1515 m/c$, Fig. 5). There are no boundary «G» and inclusions on the images. The reason is that we do not know the boundary «G». Therefore, first we have to find it.



(a) (b)

Figure 5: The image of the speed of sound: (a) standard RTM , (b) Energy RTM

We propose to obtain the image in 3 steps:

Step 1: Recovery of the boundary «G». For this purpose we use the Energy RTM process for a small time of observation $T_0 < T$ (fig. 6) (with given speed of sound in fat c_{fat}. As a result we recover the boundary «G».

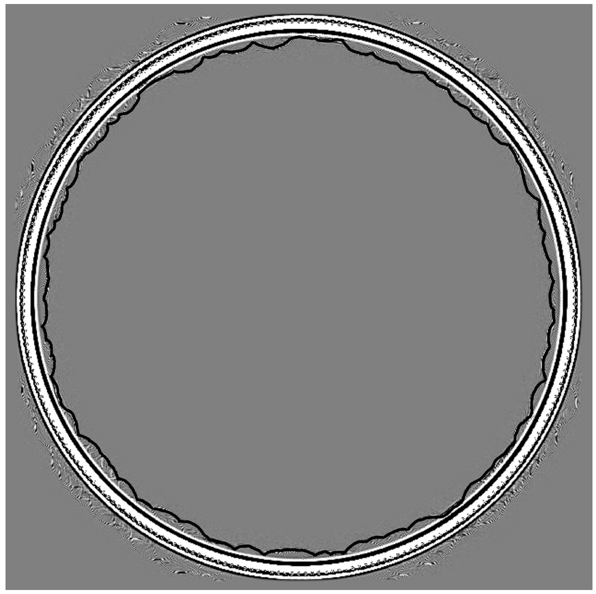


Figure 6: Image of speed of sound for a short time of observation

Step 2. Then we continue the wave field $u_0(x, t; x_s) = \frac{\partial p_0}{\partial t}(x, t; x_s)$ in reverse time. In other worlds we solve problem (4) for the speed of sound $c = c_{fat}$ in fat and $c = c_{grand}$ in glandular tissue. As a result we find the wave field $u_1(x, t; x_s)$ at the boundary Γ_1 (Fig. 7).

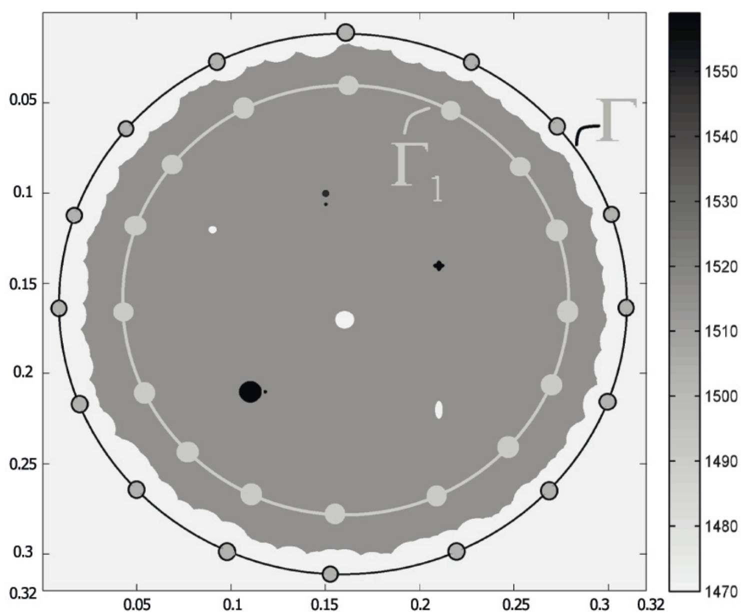


Figure 7: Virtual receivers on the circle Γ_1

Step 3. We apply Energy RTM procedure for the wave field $u_1(x, t; x_s)$ with the speed $c = c_{grand}$ in glandular tissue and obtain the final image (Fig. 8)

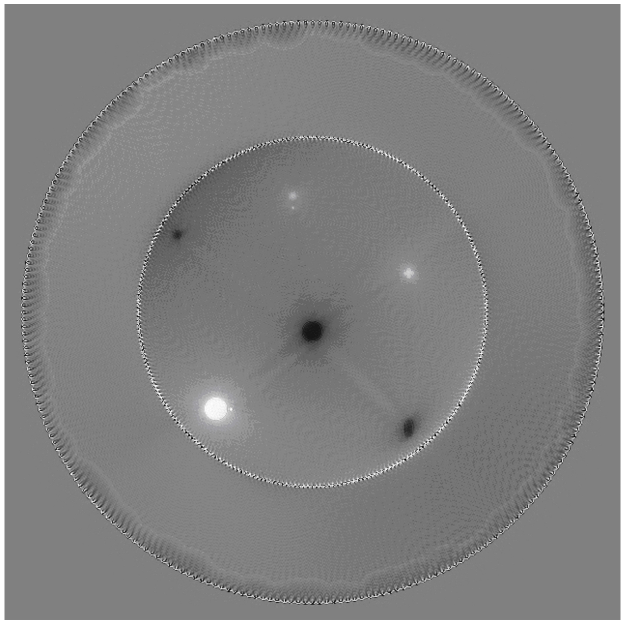


Figure 8: The final image

Notice, that the close inclusions are separated. The image obtained after standard RTM procedure is worse (Fig. 9).

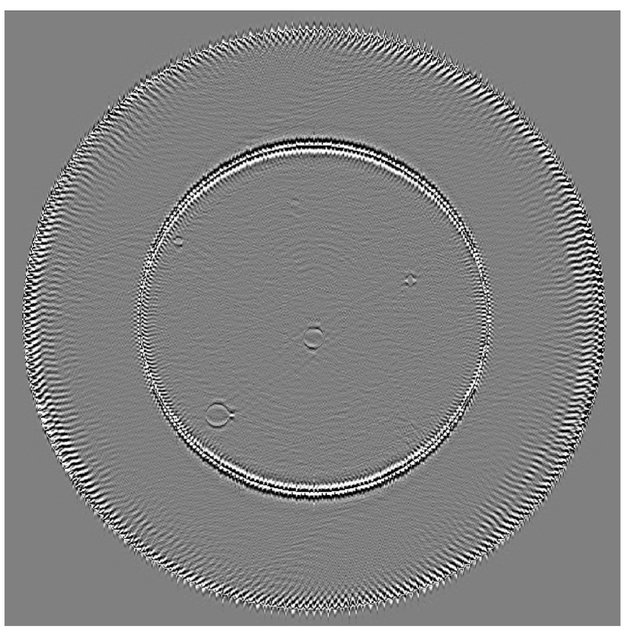


Figure 9: The image after standard RTM

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