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NUMERICAL STUDY OF NON-NEWTONIAN FLOWS WITH SOLID PARTICLES IN PIPES AND ANNULAR CHANELS

Shebelev A.V., Gavrilov A.A., Minakov A.V. [1]

Abstract The paper presents the results of modeling non-Newtonian flows with solid particles using the single-fluid model proposed by the authors. The proposed model was verified and validated on problems of steady-state turbulent flow of a non-Nontonian fluid with heavy particles in a horizontal pipe and an annular channel. Using a drilling fluid modified with carbon nanotubes as an example, the influence of fluid rheology on the reliability of modeling was studied. By comparison with experimental data, DNS-DEM modelling data and calculations performed using the two-fluid model of granular media, it is shown that the proposed model can satisfactorily predict the distribution of particle concentration and pressure drop in the channels.

Key words: non-Newtonian flow, turbulence, single-fluid model, suspension, testing.

AMS Mathematics Subject Classification: 76T20.

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1 Introduction

The study of non-Newtonian fluid flows with solid particles is of great application. This class of flows is widespread in oil and mining industries. Drilling mud used for transport and removal of drilled rock particles is a non-Newtonian fluid. Therefore, optimising drilling and mud flushing processes helps to reduce economic and environmental costs, as well as the likelihood of accidents. Large particles can be retained near the center of channel and may not settle if mud has a high yield strength [1]. Also, efficiency of particle removal generally depends on many drilling parameters such as: drilling fluid rheology, particle size and density, flow rate, drill pipe rotation speed, etc. The description of cuttings transport in turbulent flow regime is even more difficult task due to stochastic nature of fluid turbulence and its interaction with solid particles.

In this connection, methods of numerical modelling of two-phase flows with dispersed particles are being actively developed. The approach of direct numerical modelling of DNS with resolution of flowing of each particle in flow is more and more often used for modelling suspensions. However, for applied problems, it is inexpedient to use this approach because of extremely high computational cost of the DNS method. However, for research purposes, such calculations make a significant contribution to fundamental understanding of flow-particle interactions in suspensions.

The full Eulerian approach is also widely used to model flow of suspensions. Because of its computational efficiency, this model is often used for modelling particle transport. The use of two-fluid approach gives good agreement of solid phase velocity distribution with experimental data. However, one of the main problems of two-liquid approach is that to close the equations for solid phase, relationships for drag force, lift force, particle collision mechanism, etc. need to be defined. These relations can be very complex to model and must be calibrated for each

[1]Corresponding Author.

class of flows.This paper presents the results of testing the single-fluid model of a two-phase turbulent non-Newtonian flow with coarse particles proposed by the authors.

2 Mathematical model

In [2, 3, 4], a model for laminar and turbulent flows was proposed and tested based on conservation laws for mixture as a whole, assuming a quasi-equilibrium sliding velocity of phases. The peculiarity of model in comparison with two-fluid models is solution of a system of equations for a two-phase flow as a whole using rheological relations and a transport equation for concentration of dispersed phase with an algebraic equation for interphase slip rate. The rheological model describes viscous and contact modes of particle motion in a liquid.

The approach to describing dispersed phase is associated with introduction of a set of continua, each of which refers to a particular phase of mixture. The quantities referring to solid disperse and carrier liquid phases are denoted by the lower indices p and f, respectively. The volume concentration of particles ϕ characterises the proportion of the volume occupied by the dispersed phase. Non-Brownian particles are assumed to be spheres with constant diameter d_p and constant density ρ_p. Averaging of equations describing turbulent flow of a suspension is carried out using the Favre approach or phase averaging [2].

The rheological model of turbulent suspension flow includes equations of continuity, quantity of motion and turbulent transport formulated for the mixture, as well as a transport equation for particle concentration and an algebraic equation for relative interfacial velocity. The Favre averaged mass and momentum balance equations of mixture are of the form:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \tag{1}$$

$$\rho \frac{d\mathbf{U}}{dt} = -\nabla p + \rho \mathbf{g} + \nabla \cdot (\mathbf{T_v} + \mathbf{T_t}), \tag{2}$$

where p is averaged pressure, T_t is Reynolds stress tensor of mixture, T_v is viscous stress tensor of mixture. The turbulent stress tensor is modeled using the Boussinesq hypothesis:

$$T_t = 2\rho\nu_t \left(S - \frac{1}{3} (\nabla \cdot \mathbf{U})\,\mathrm{I} \right) - \frac{2}{3}\rho k \mathrm{I}, \tag{3}$$

where k is the kinetic energy of turbulent pulsations, $S = 0.5(\nabla\mathbf{U} + \nabla\mathbf{U}^{\mathbf{T}})$ is the tensor of the averaged motion deformation rates, and $\mathbf{I}$ is the unit tensor. The turbulent flow of particle concentration is closed using the gradient hypothesis:

$$\mathbf{u}_d = -\frac{1}{\phi(1-\phi)} \frac{\nu_t}{\sigma_\phi} \nabla\phi, \tag{4}$$

where ν_t is the turbulent viscosity, σ_ϕ is the turbulent Schmidt number.

The paper uses a turbulence model based on the eddy viscosity hypothesis: a two-parameter model with an equation for the specific dissipation rate $k - \omega$ SST [5]. In the mass and momentum balance equations of the two-phase medium, the terms describing the interphase interaction are absent.

The Favre averaged particle concentration transport equation has the form:

$$\frac{\partial \rho_p \phi}{\partial t} + \nabla \cdot \left[\rho_p \phi \left(\mathbf{U} + \frac{\rho_f}{\rho} (1-\phi)\, \mathbf{U}_r \right) \right] = 0 \tag{5}$$

The equation for the velocity of a continuum of particles simplifies to the algebraic equation for the relative velocity $\mathbf{U}_r = \mathbf{U}_p - \mathbf{U}_f$:

$$\mathbf{U}_r = \frac{\tau_p}{\rho_p} \left\{ (\rho_p - \rho_f) \left[\mathbf{g} - \left(\frac{\partial \mathbf{U}}{\partial t} + (\mathbf{U} \cdot \nabla)\,\mathbf{U} \right) + \nabla \cdot T_t \right] + \frac{\mathbf{F}_L - \nabla \Pi_p}{\phi} \right\} + \mathbf{u}_d, \tag{6}$$

where τ_p is time of dynamic relaxation of a particle in a constrained flow, $\mathbf{u}_d$ is diffusion velocity arising from difference between the averaged fluid velocity and the averaged fluid velocity along particle trajectory (so-called fluid velocity visible to particle), $\mathbf{F}_L$ is Saffman lift force in shear flow and Π_p is normal tension of particle continuum.

To describe mesoscale level of flow, rheological approach with introduction of phenomenological algebraic dependences of normal and tangential stresses on particle concentration and velocity gradients, described in detail in [2], is used.

Herschel-Bulkley rheology [6] was used to describe the non-Newtonian properties of the fluid, where the local average effective viscosity is calculated from the average strain rate in the fluid as:

$$\mu_f = \left(\tau_y + k_v \dot{\gamma}_f^n\right)/\dot{\gamma}_f, \tag{7}$$

here: τ_y is ultimate stress, k_v is consistency index, $\dot{\gamma}_f$ is average strain rate in liquid and n is flow behavior index.

3 Steady turbulent flow in a circular pipe

The steady turbulent flow of a suspension in a round pipe is considered [7]. The computational domain is a circular pipe with a diameter of D=0.044 m. The density of drilling fluid was $\rho = 1000\,\mathrm{kg/m^3}$. The rheology of drilling fluid was set from experimental data (see Tab. 1), where Re_g is Reynolds number, U_m is mixture velocity, C_v is volume concentration, τ is ultimate stress, k is consistency index, n is flow behavior index. To describe the non-Newtonian properties of liquid, the Herschel-Bulkley rheological model was used. Spherical particles with a size $d = 2$ mm and a density $\rho = 2600\,\mathrm{kg/m^3}$ were considered as sludge particles. The gravity vector was directed perpendicular to the pipe axis.

Table 1: Flow condition [7]

Re_g	U_m, m/s	C_v	τ,Pa	k, $Pa \cdot s^n$	n
8500	3.11	0.1	0.23	0.479	0.58

The σFlow in-home software package [8, 9] was used for numerical modeling. To solve the problem, an unstructured grid was constructed, consisting of $60 \times 100 \times 2$ (60 along the radius and 100 along the circumference and 2 along the channel length) computational nodes, with condensation towards the walls, based on the condition of the dimensionless distance to first wall node $y^+ \sim 1$. Since the problem statement implies a steady flow ($\partial p/\partial z =$ const, $\partial \mathbf{U}/\partial z = 0$, $\partial \phi/\partial z = 0$), only 2 cells were constructed along channel. On the surface of the channel wall, the no-slip conditions and the absence of particle flow are satisfied ($\mathbf{U} = \mathbf{U}_w$, $\mathbf{U}_w = 0$, $\partial \phi/\partial n = 0$).

A calculation was carried out for the liquid from Table 1 for a Reynolds number of 8500. Below (Fig. 1) is a graph of the distribution of particle concentration over the height of the pipe.

The rheological model gives a good agreement of the concentration profile distribution along the channel height with the experiment [7] and the DNS-DEM calculation [7], but overestimates the concentration on the lower wall (Fig. 1). For the experiment and DNS-DEM calculation, the region with increased particle concentration near the lower wall has a greater height, about 0.2 of the channel height, while the calculation in σFlow is only about 0.1 of the channel height. Fig. 2(left) shows the particle distributions in the cross section.

Fig. 2 (right) shows the distribution of the longitudinal velocity component in the cross section of the channel. In Fig. 2 (right), the flow core is shifted to the upper edge, which is due to the filling of the channel with particles in its lower part.

Tab. 2 shows that the proposed model gives a good agreement for the pressure drop, the difference is 2%. The ratio of C_d/C_v value shows ratio of delivered concentration to volumetric

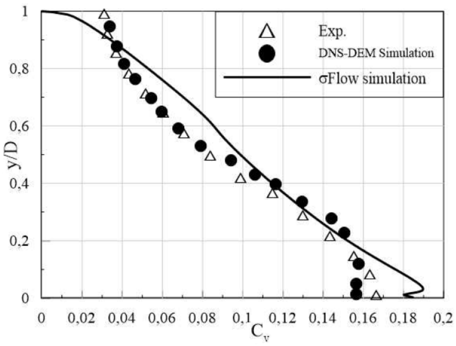

Figure 1: Distribution of particle concentration along the channel height for a Reynolds number of 8500.

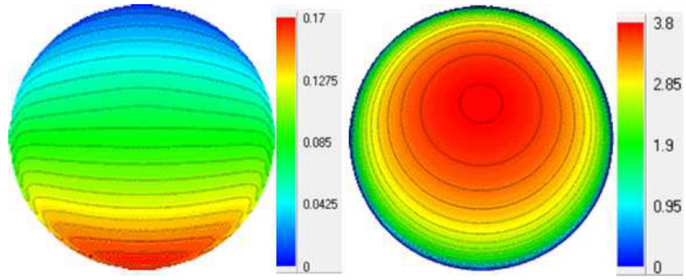

Figure 2: Distribution of particle concentration (left) and longitudinal velocity component (right) in cross section for a Reynolds number of 8500.

concentration; for this case, this value is 1, which means that almost all particles are removed from channel.

Table 2: Integral results, model $k-\omega$ SST

Re_g	C_v, m/s	dp/dz, kPa, exp.	dp/dz, kPa	C_d/C_v
8500	0.1	4.6	4.5	0.986

4 Steady turbulent flow in an annular channel

Further, to test developed mathematical model, flow with solid particles in a horizontal well of hydrocarbon-based drilling mud modified with multi-walled carbon nanotubes was considered. A detailed experimental study of characteristics of drilling fluid modified by carbon nanotubes is given in [10]. To test developed model, similar calculations were carried out in parallel in the Fluent [11] software package. The Eulerian model with granular media was used in the Fluent software package.

The σFlow software was used for numerical modelling. A computational grid consisting of $50 \times 80 \times 100$ (50 on radius and 80 on circumference and 100 on length of channel) computational nodes was used, grid was densified to channel walls, based on condition of dimensionless distance to the first near-wall node $y+1$. Preliminary methodological calculations showed that this number of nodes is sufficient to obtain a numerical solution that does not depend on further detailing of the grid. The length of the design domain of the annular channel was set to 10 m. This length was sufficient to establish the flow and sludge concentration along the length of channel. The same grid was built in the Fluent software package.

The calculation domain is an annular channel formed by two smooth straight tubes of circular cross-section (Fig. 3). The inner pipe rotates around its axis with constant angular velocity. To calculate flows in the borehole when pumping the modified drilling fluid, typical parameters of directional drilling process are chosen. The diameter of inner pipe is $D_1 = 0.127$ m, diameter of outer pipe is $D_2 = 0.2207$ m. The rotation speed of the drill pipe was equal $\omega = 40$ rpm, speed of the mixture at inlet was $U_m = 2$ m/s. The density of drilling mud was equal $\rho = 968\,\mathrm{kg/m^3}$. The rheology of drilling fluids was set from the experimental data (Tab. 3). Non-Newtonian properties are described using the Herschel-Bulkley model.

Spherical particles with a size $d = 1$ mm and a density $\rho = 3000\,\mathrm{kg/m^3}$ were considered as sludge particles. The concentration of cuttings at channel inlet was set $C_v = 3\%$ by volume. The horizontal section of the well was considered, as horizontal sections are the most difficult in drilling process and are subject to accumulation of cuttings particles during their transport.

Table 3: Experimental data [10]

Concentration of MWCNT, by mass,%	τ,Pa	k, $Pa \cdot s^n$	n
0	2.879	0.4411	0.5381
0.1	3.475	0.5026	0.5503
0.25	4.114	0.6072	0.5394
0.5	4.228	0.6313	0.5302

The gravity vector was directed perpendicular to the axis of the annular channel.

The two-parameter turbulence model $k - \omega$ SST was used to describe turbulent characteristics of the flow. At the outlet of the annular channel the condition of free exit with a fixed value of pressure ($p = 0$, $\partial \mathbf{U}/\partial z = 0$, $\partial \phi/\partial z = 0$) was specified. On the pipe surfaces, non-slip conditions and the absence of particle flow ($\mathbf{U} = \mathbf{U}_w$, $\partial \phi/\partial n = 0$) were satisfied. On the inner wall of the annular channel, the angular velocity value of rotation ($u_{w,\tau} = \omega R_1$, $u_{w,n} = 0$) was specified. The outer wall was at rest ($\mathbf{U}_w = 0$).

Fig. 4 shows dependence of pressure drop as a function of MWCNT concentration. The data obtained in different software packages are in good agreement with each other. The error is up to 5%. It can be seen that pressure drop increases with the increase of MWCNT

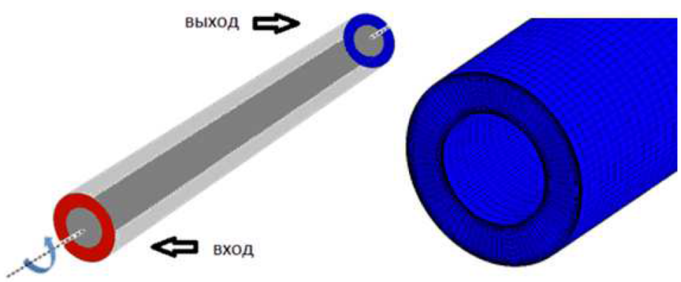

Figure 3: Schematic representation of the geometry of the computational domain (left) and a fragment of the computational mesh (right).

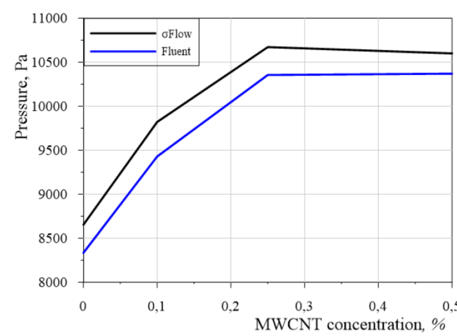

Figure 4: Pressure drop depending on MWCNT concentration for the σFlow and Fluent software packages.

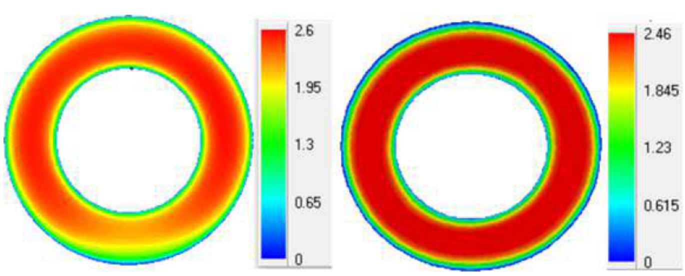

Figure 5: Axial velocity of drilling fluid at out from annular channel for mass concentration of MWCNTs in drilling fluid of 0%, calculation of σFlow on the left, Fluent on the right.

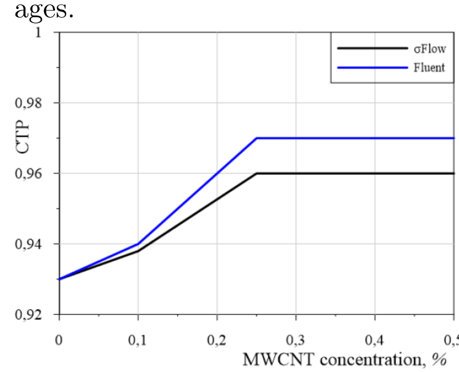

Figure 6: Dependence of sludge removal efficiency coefficient on the MWCNT concentration for the σFlow and Fluent software packages.

concentration in drilling fluid. The comparison of the axial velocity distribution of the drilling fluid at the exit of the annulus for the mass concentration of MWCNTs in the drilling fluid of 0% for σFlow and Fluent is presented in Fig. 5.

Fig. 5 shows that when calculating in σFlow, the velocity near the bottom wall is slightly lower than in the upper part of the annular channel, which indicates an increased concentration of particles there. For calculations in Fluent, the velocity profiles are more symmetrical in the lower and upper parts of the well. This is due to more intense migration of particles into the flow core.

Fig. 6 shows dependence of sludge removal efficiency coefficient on the MWCNT concentration. The sludge removal efficiency coefficient was calculated as ratio of volume-averaged fluid velocity to volume-averaged particle velocity. The data are in good agreement with each other. The error is up to 1%. It can be seen that with an increase in concentration of MWCNTs in drilling fluid, cuttings removal efficiency coefficient increases. When 0.25% MWCNT is added to drilling fluid, cuttings removal efficiency increases by 3% and pressure drop increases by 20%.

Conclusion

The continuum model of two-phase flows of suspensions for carrier Newtonian fluids, governed by the Herschel-Bulkley rheological model, presented in [2, 3, 4], is developed for the case of flows of non-Newtonian fluids with solid particles, taking into account the features that arise at the mesoscopic level of description of the flow of fluid with solid particles.

The mathematical model and software for description of turbulent multiphase flow during the transport of sludge particles have been tested on problem of turbulent flow of non-Newtonian fluid in a circular pipe with sludge particles of 2 mm diameter. The results of numerical simulation obtained with the developed model of sludge particle transport were compared with the data of field experiment [7] and data of DNS - DEM modelling (Direct Numerical Simulation - Discrete Element Method) [7]. Comparison was carried out with data on pressure drop in the channel and the shape of particle concentration profiles.

To test developed mathematical models, study of cuttings particles transport in a horizontal well by hydrocarbon-based drilling mud modified with multi-walled carbon nanotubes was carried out. To test the proposed models and algorithms of cuttings particle transport, similar calculations were carried out in parallel in the ANSYS Fluent software package using the Euler model with granular medium. In both packages the influence of MWCNT concentration on flow characteristics and efficiency of cuttings removal from the well was investigated. As a result of cross-verification of the packages, it was shown that the software packages give generally quite similar results. The difference in the results of calculations for pressure drop is about 5%, the difference in the integral efficiency of cuttings removal from the annular channel is about 1%. For this problem it is established that when 0.25% MWCNT is added to the drilling fluid, the mud removal efficiency increases by 3%, and the pressure drop increases by 20%.

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Shebelev A.V.,
Siberian Federal University,
Svobodnyi avenue 79, Krasnoyarsk, Russia,
Kutateladze Institute of Thermophysics SB RAS
Lavrentieva Av., 1, Novosibirsk, Russia
Email: `aleksandr-shebelev@mail.ru`,

Gavrilov A.A.,
Siberian Federal University,
Svobodnyi avenue 79, Krasnoyarsk, Russia,
Kutateladze Institute of Thermophysics SB RAS
Lavrentieva Av., 1, Novosibirsk, Russia
Email: `gavand@yandex.ru`,

Minakov A.V.,
Siberian Federal University,
Svobodnyi avenue 79, Krasnoyarsk, Russia,
Email: `aminakov@sfu-kras.ru`

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