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MELTING OF CLOUDY ICE UNDER IRRADIATION BY AN ARTIFICIAL RADIATION SOURCE

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Abstract The problem of melting a layer of ice with bubbles containing radiation-absorbing gas was carried out using a numerical method. The problem statement is radiation-conductive heat exchange in a semitransparent two-phase medium that selectively absorbs radiation with a first-order phase transition. The radiation transfer equation was solved by a modified mean flux method, taking into account a wide range of optical properties of the two-phase medium and the radiation source. The melting rate and growth of non-irradiating boundary depending on various optical parameters of the medium are calculated. The significant influence of anisotropic scattering and strong absorption of radiation by gas on the heating and melting of the ice layer is shown.

Key words: Cloudy ice, melting, bubbles, absorption of radiation by gas, anisotropic scattering.

AMS Mathematics Subject Classification: 80-10, 80A05, 80A21, 80A22.

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1 Introduction

Natural ice may contain various inclusions, including water vapor, carbon dioxide, methane, and other gases [1] - [2], which absorb heat radiation. In our work, we consider cloud ice [1] is a two-phase translucent medium. Studying the properties of such ice is of interest, since the gases contained in it may be greenhouse gases. In [3], [4], [5], the authors mainly considered ice as a light-scattering medium and neglected the effect of radiation absorption by bubble gas contained in ice. In [6], radiative-conductive heat transfer with a first-order phase transition in a snowpack containing solid inclusions - soot, depending on the absorption coefficient, was considered. In [7], a mathematical model of ice heating and melting in the high-altitude lake Ngoringo in the Xinjiang Tibetan Plateau (Peoples Republic of China) is presented. The presented model showed good agreement with the results of field observations and the authors recommend using the model to predict the ice state of rivers, lakes and sea ice. At the same time, the model does not take into account the volumetric absorption of radiation by various inclusions. In this paper, the optical properties of the gas phase on the melting the layer of the ice are investigated.

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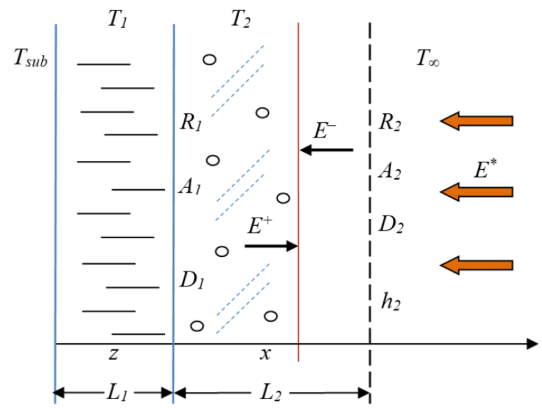

Figure 1: Geometric scheme of the problem.

2 Problem statement

This work is a continuation of our research, which uses the approach described in [4], [5]. The formulation of the problem corresponds to the experimental conditions created in [8] (Fig. 1). The constant temperature T_∞ supported into a climate control equipment. The layer of ice is adhered on an opaque, vertical bakelite substrate and irradiated by source with a filament temperature of 3200 K. Ice is an optical medium in which radiation is selectively absorbed and anisotropically scattered in a translucent medium. We assume that all bubbles are spherical with an average radius of r_b and that they are uniformly distributed over the volume with f_V for simplicity. The gas in the bubbles selectively absorbs radiation. The selectivity of the radiation is modeled by three bands of the absorption spectrum. Tab. 1 presents the spectral properties of ice [1], the radiation source, and the absorption coefficients of the gas. Optical properties of ice layer boundaries are presented in [4] - [5].

The solution to the problem is divided into two stages and is described in [4]-[5].

The non-stationary energy equations for the substrate with temperature $T_1(z,t)$ and the ice layer with temperature $T_2(x,t)$, taking into account the content of the gas phase in the bubbles, are written as follows [4]:

$$\begin{aligned} c_1\rho_1\frac{\partial T_1(z,t)}{\partial z} &= \lambda_1\frac{\partial^2 T_1(z,t)}{\partial z^2} \\ c_2\rho_2\frac{\partial T_2(x,t)}{\partial x} &= \frac{\partial}{\partial x}\left(\lambda_2\frac{\partial T_2(x,t)}{\partial x} - E_\nu(x,t)\right) \end{aligned} \tag{1}$$

Here c_i is the heat capacity, ρ_i is the density, λ_i is the coefficient of thermal conductivity, a_i is the thermal diffusivity ($i = 1, 2$, substrate and ice, respectively). $\lambda_2 =$

Table 1: The Values of spectral parameters of absorption coefficient, parameter of scattering of ice and the radiation source.

j	λ_j, μm	α_j, m^{-1}	$\alpha_{gas,j}$, m^{-1}	E_j^*,W m^{-2}
1	0.33 – 0.75	2.8	0; 1.5; 50	645
2	0.75 – 1.5	10	0; 1.5; 50	2151
3	1.5 – 3.0	450	0; 1.5; 50	1371

$(1-f_V)\,\lambda_{ice}+f_V\lambda_{gas}$ is the effective thermal conductivity of ice, where λ_{ice} and λ_{gas} are the thermal conductivity of pure ice and gas, respectively. $c_2=(1-f_V)\,c_{ice}+f_V c_{gas}$ is the effective heat capacity of ice, where c_{ice} and c_{gas} are the heat capacity of ice and gas, respectively. $E(x,t)=E^{+}(x,t)-E^{-}(x,t)$ is the density of the resulting radiation flux.

Boundary conditions for the ice heating stage:

$$\begin{aligned} T_1(z,t) &= T_{sub}, \quad z=0,\\ -\lambda_1\frac{\partial T_1(z,t)}{\partial z} &= -\lambda_2\frac{\partial T_2(x,t)}{\partial x}+A_1E^{-}(x,t), \quad z=L_1, \quad x=0,\\ -\lambda_2\frac{\partial T_2(x,t)}{\partial x} &= h(T_2-T_\infty)+|E_{res}|, \quad x=L_2. \end{aligned} \tag{2}$$

Here h is the heat transfer coefficient, $|E_{res}|=A_2\,(E^{+}+E^{*})-\varepsilon_2\sigma_0^4\,(T_2^4-T_\infty^4)$, σ_0 is the Stefan-Boltzmann constant. Eqs. (1) and (2) are supplemented by the initial condition: $T_1(z,0)=T_2(x,0)=T_{sub}$.

At the phase transition stage, the boundary conditions (2) for the substrate and the left ice surface do not change, the temperature of the irradiated boundary $T_2(L_2(t),t)=T_f$ is constant. The boundary condition of the right surface is transformed into the Stefan condition, taking into account the formed water film:

$$\lambda_2\frac{\partial T_2(x,t)}{\partial x}+h(T_{fil}-T_\infty)+|E_{res,fil}|=\rho_2\gamma\frac{\partial L_2(t)}{\partial t}, \quad x=L_2(t). \tag{3}$$

Here $|E_{res,fil}|=A_2\,(E^{+}+E^{*})-\varepsilon_2\sigma_0^4\left(T_2^4-T_{fil}^4-T_\infty^4\right)$, γ is latent heat of the phase transition. Eq. (1) at the second stage is supplemented by the initial condition: $T_2(x,0)=q(x)$ and $L_2(0)/L_2=1$.

The resulting radiation flux densities $E^{\pm}(x,t)$, $E_\nu=\sum_j\left(E_j^{+}-E_j^{-}\right)$, included in Eqs. (1)-(3) are solved by the differential modified mean flux method, first used to solve neutron transport in nuclear reactors [9] (j is the number of the spectral band). In this method the integral-differential equation of radiation transfer is reduced to a system of two nonlinear differential equations for a plane layer of a semitransparent absorbing and anisotropic radiation scattering medium are represented in the form [9]-[10]:

$$\begin{aligned} \frac{d}{d\tau_j}\left(E_j^{+}-E_j^{-}\right)+(1-\omega_{j,tr})\left(m_j^{+}E_j^{+}-m_j^{-}E_j^{-}\right) &= (1-\omega_{j,tr})\,n^2B_j,\\ \frac{d}{d\tau_j}\left(m_j^{+}l_j^{+}E_j^{+}-m_j^{-}l_j^{-}E_j^{-}\right)+(1-\omega_{j,tr})\left(E_j^{+}-E_j^{-}\right) &= 0. \end{aligned} \tag{4}$$

The boundary conditions are as:

$$\begin{aligned} E_j^{-} &= \frac{1}{4}\,\varepsilon_1B_j+R_1E_j^{+}, \qquad \tau_{j,tr}=0,\\ E_j^{+} &= (1-R_2)\,E_j^{*}+\left(1-\frac{n^{*2}}{n^2}\,(1-R_2)\right)E_j^{-}, \qquad \tau_{j,tr}=\tau_j. \end{aligned} \tag{5}$$

Here B_j is the Planck function, $\omega_{j,tr}=\beta_{j,tr}/\kappa_{j,tr}$ is the transport albedo of the single scattering; n is the refractive index of the medium; n^{*} is the refractive index of the ambient air, $\beta_{j,tr}$ is the transport scattering coefficient; α_j is the absorption coefficient; $\tau_{j,tr}=\kappa_{j,tr}L_2(t)$ is the optical thickness. The values of the coefficients $m^{\pm}$ and $l^{\pm}$ are determined from the recurrence relation obtained using the formal solution of the radiation transfer equation [9].

To take into account anisotropic scattering of radiation from spherical particles with average radius much larger than the wavelength of the incident radiation, $r_b \gg \lambda_j$, we use the method of transport approximation [3], [6]-[7]:

$$\beta_{tr} = 0.675\,(n-1)\,z, \qquad z = f_V/r_b. \tag{6}$$

Eq. (1) with boundary conditions (2)-(3) and Eq. (4) with boundary conditions (5), taking into account (6), are reduced to a dimensionless form in the same way as it is done in [11]. The solution to the problem consists in determining the dynamics of melting and temperature increase of the non-irradiated ice surface depending on time.

3 Discussion

The following is an analysis of the results of numerical calculation of the substrate-ice system with the following physical parameters: substrate thickness $L_1 = 0.015$ m, initial ice thickness $L_2 = 0.045$ m, temperature of the left boundary of the substrate and initial ice temperature $T_{sub} = T_2(x,0) = 260$ K, constant atmospheric temperature inside the chamber and ice melting temperature $T_f = T_\infty = 273$ K. Thermophysical properties of bakelite substrate and ice: thermal conductivity $\lambda_1 = 0.232$ W/(m K), $\lambda_2 = 1.88$ W/(m K); heat capacity $c_1 = 1590$ J/(kg K), $c_2 = 2200$ J/(kg K); thermal diffusivity $a_1 = 1.1 \cdot 10^{-7}$ m^2/s, $a_1 = 1.04 \cdot 10^{-6}$ m^2/s [8]; latent the heat of the phase transition $\gamma = 335$ kJ/kg. Optical parameters: refractive index of ice $n = 1$ (for air $n^* = 1$), emissivity of the right irradiated surface $\varepsilon_2 = 0.115$, left reflection coefficients $R_1 = 0.96$ and $R_2 = 1 - \varepsilon_2$, emissivity of the left boundary $\varepsilon_1 = 1 - R_1$. The spectral characteristics of the ice and the radiation source are presented in Tab. 1. α_{gas}, z, r_b were varied in the calculations.

Figs. 2 and 3 show the growth rate of the temperature of the non-irradiated surface and the dynamics of melting of the ice layer. Fig. 2 shows the temperature growth rate of the non-irradiated surface of the ice layer for different scattering parameters z and r_b and different $\alpha_{gas,j}$. We used $z = 10$ and 30 m^{-1} and r_b=10^{-4}, 10^{-3} and 10^{-2} m in the calculations. The graphs show that the rate of temperature growth depends more strongly on the average radius of the bubbles. The z parameter does not greatly affect the temperature increase, however, an increase in both parameters, and therefore an increased value of the gas phase f_V, leads to a general decrease in temperature at the beginning of the melting stage. This can be explained by a decrease in the effective thermal conductivity and heat capacity of the condensed phase and absorption of the gas phase closer to the right boundary.

The rate of melting of the ice layer (Fig. 3) is practically independent of optical parameters due to the small thickness of the layer; in the initial position it is 4.5 cm.

Fig. 4 shows the results of calculations on the influence of the spectral absorption band on the heating and melting of the ice layer depending on $\alpha_{gas,j} = 50$ m^{-1} and with scattering parameters $z = 30$ m^{-1} and $r_b = 10^{-3}$ m. Fig. 4a shows the temperature field at the final point of the heating stage calculations. As can be seen from the graph, taking into account the gas absorption coefficient in the most absorbing part of the spectrum of the ice layer, j=3, does not lead to strong changes in temperature, the maximum difference reaches $\sim$ 0.02 Celsius. Obviously, this circumstance does not affect the rate of ice melting (Fig. 4b). The reason lies in the spectrum of the artificial

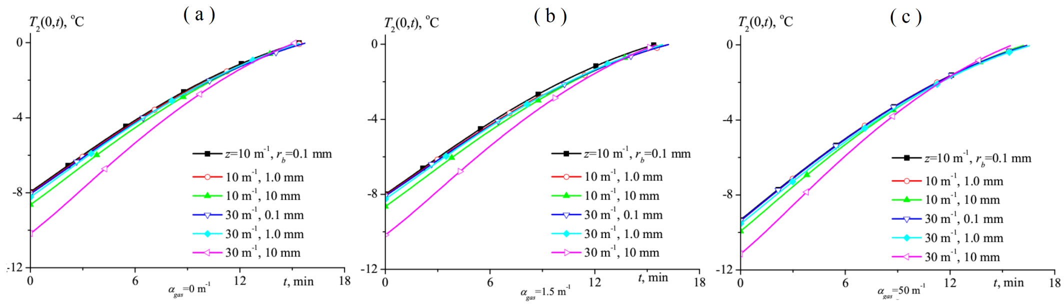

Figure 2: Dynamics of temperature growth of the left non-irradiated boundary of the ice layer in the melting stage at different values of the scattering parameters and α_{gas} (a is the $\alpha_{gas} = 0$, b is the $\alpha_{gas} = 1.5m^{-1}$, c is the $\alpha_{gas} = 50m^{-1}$).

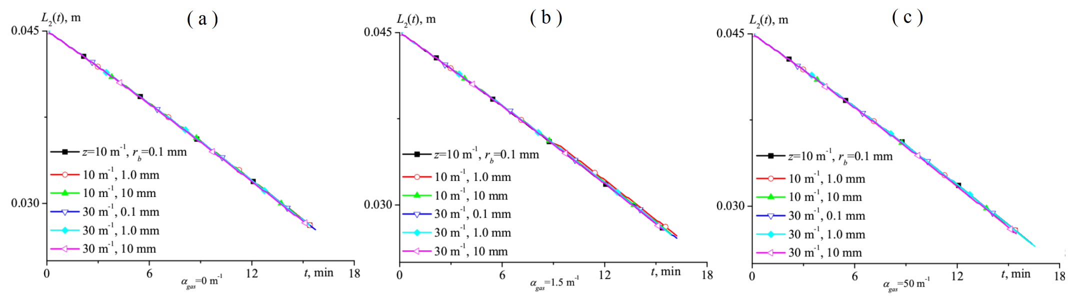

Figure 3: Dynamics of melting of the cloudy ice layer at different values of the anisotropic scattering parameters and α_{gas} (See nomenclature in Fig. 2).

radiation source, where most of the total incident radiation falls on the “transparency windows” of the ice, and a smaller part in the IR-range (1.5-3 μm) and where ice has maximum absorption (see Tab. 1). Radiation falling on a layer of ice is most effectively absorbed and scattered in the spectral regions j=1 and 2, heats and subsequently melts the ice, while in j=3 the incident radiation does not enter the volume of the medium and is absorbed mainly on the surface. If the bubbles contain a gas with an absorption spectrum in the j=3 range, for example, methane, it will not have a strong effect on the melting of ice, but will still be released into the atmosphere, where it will become part of the greenhouse gases.

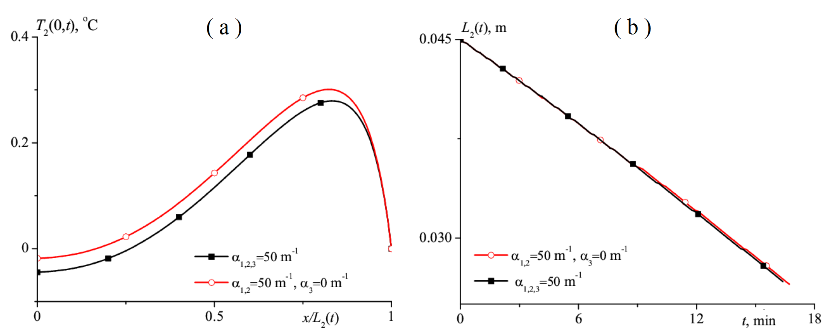

Figure 4: The temperature field in the ice layer at the end of melting (a) and the melting rate (b) of the ice layer, depending on the $\alpha_{gas,j}$.

4 Conclusion

The numerical calculation of the melting dynamics and thermal state of the cloudy ice layer was performed using mathematical modeling methods. In the problem statement, the ice layer is represented as a two-phase translucent medium. To solve the radiation part, the method of mean fluxes with three absorption bands was used; anisotropic scattering was taken into account using the transport approximation method. It is shown that weak absorption of radiation from the model gas does not greatly affect the heating and melting of the ice layer. Melting is more influenced by the strong absorption of radiation by the gas and the average radius of the bubbles. There is no need to take into account the absorption of radiation by gas in the IR range when irradiating ice with a high-temperature radiation source. The calculation results can be used by various monitoring services.

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